Experiment # 3: Vector Addition Using the Force Table

Object: To learn the rules of vector addition, both graphical and analytical.

Introduction.

Scalars and Vectors: Many physical quantities need a single number, plus the appropriate units being used in the measurement; such quantities are known as *scalars*. Examples are temperature, mass, energy, density, etc. When speaking of a *scalar* we will use a plain letter or a combination of letters and numbers: T, m_1, KE, etc.
Vectors in two dimensions: Other physical quantities require two numbers (in two dimensions) or three numbers (in three dimensions) to be uniquely determined: they are known as *vectors* ; here we will use a boldface-type letter to indicate that a quantity is a vector, **a**, **A**, **F**, **d** ; when hand writings, people use a regular letter with an arrow, or half-arrow on top of it . Mathematically, *a vector is an ordered set of numbers*.
In this lab we will limit our studies to two-dimensional vectors, so a vector will be uniquely determined if you give two numbers. There are many ways to do this, and the most popular ways of doing this are by giving,

i) its components, $\boldsymbol{a} = (a_x , a_y)$, where a_x is known as the x-component , and a_y as the y-component. Being an ordered set, the first number always represents the x-component, while the second number represents the y-component. This is known as the rectangular or Cartesian form of a vector in two dimensions. The graphical representation of a vector in rectangular coordinates is an arrow whose tail is at the origin and its head, or tip, is at the point with coordinates (a_x , a_y)

ii) its magnitude a and direction θ_a ; this is known as the Polar representation of a vector. Graphically a vector is an arrow of length a, the arrow making an angle θ_a , angle is measured starting at the + x axis line and ending at arrow, angle measured in the counterclockwise way.

Using geometry and trigonometry, we can show that $a = (a_x^2 + a_y^2)^{½}$, and $\tan \theta_a = a_y / a_x$. [Note that this means that you must be careful when determining θ_a . If $a_x > 0$, then $\theta_a = atan (a_y / a_x)$; but if $a_x < 0$, then $\theta_a = atan (a_y / a_x) + 180^o$.]

Similarly, we can show that $a_x = a \cos \theta_a$, and $a_y = a \sin \theta_a$

Fig.1a illustrates for us a vector representing a force exerted on a chair by a student, $\boldsymbol{F_1}$, having a magnitude of 3.0 N [Here N stands for 'Newtons', the metric unit of force] in the direction 30º North of West. For graphical purposes, we must convert the magnitudes provided into arrows whose length fit the surface where we will draw the arrow (find it convenient to use a scale that allows us to sketch the arrows on a piece of paper of practical size, most commonly 8.5" x 11" , or a writing board in a classroom or office).

In our example we use 1.0 N = 2.0 cm, with East , or $\theta = 0^o$, assumed to be along the positive *x*-axis; North, or $\theta = 90^o$, assumed to be along the positive y-axis, etc.
We represent the vector as a 6.0 cm arrow pointing in a direction making an angle of 150º with the positive x-axis. The magnitude of the force vector is $F_1 =$ 3.0 N and its direction is $\theta_1 = 150^o$.
Its components are given by

$F_{1x} = F_1 \cos\theta_1 = 3.0 \cos 150^o = -2.60$ N $\qquad$ $F_{1y} = F_1 \sin\theta_1 = 3.0 \sin 150^o = 1.50$ N

Graphically we can find the components by determining the projection of the arrow along the x and y axes respectively, Fig. 1b. Either way, we find that $\boldsymbol{F_1}$ = (-2.60 , 1.50) N.

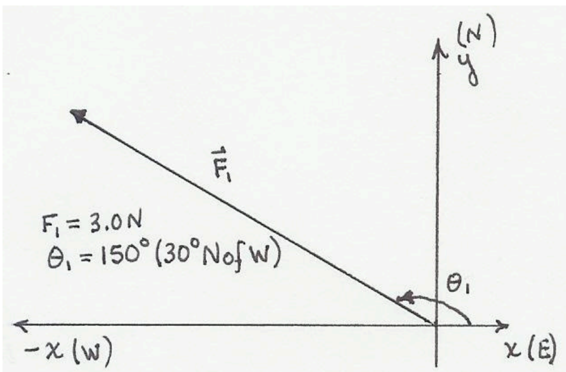


Fig 1a. A vector drawn to scale

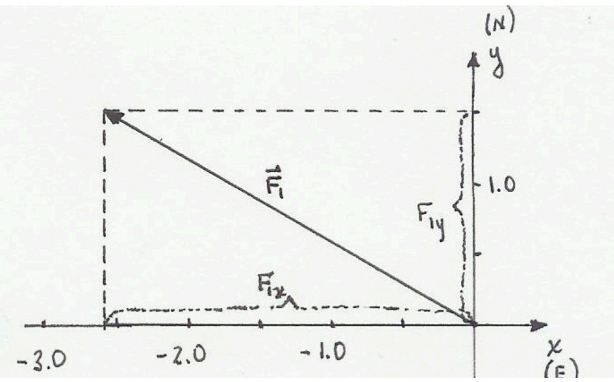


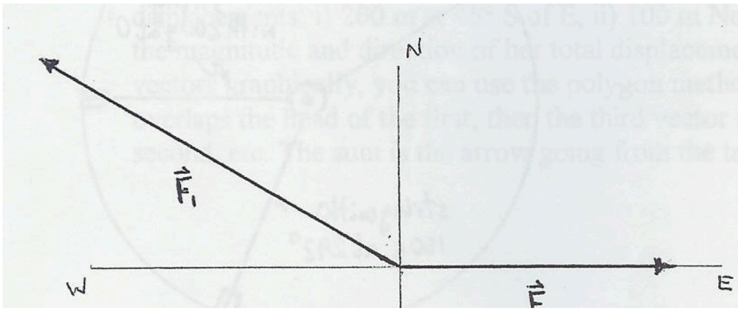
Fig. 1b. The vector and its components

Vector Addition.- Suppose that the chair in the previous example is being simultaneously pushed by a second student exerting a force $\boldsymbol{F_2}$, with magnitude 2.0 N and pointing to the East (So here we have $F_2 = 2.0$ N and $\theta_2 = 0^o$ or $F_{2x} = 2.0$ N and $F_{2y} = 0.0$ N); then the net effect of both forces when acting simultaneously is found by doing a *vector addition* . We denote the vector sum as $\boldsymbol{F_1} + \boldsymbol{F_2}$, and refer to it as the "net force", $\boldsymbol{F_{net}} = \boldsymbol{F_1} + \boldsymbol{F_2}$.

Analytically, the addition is a vector with components equal to the sum of the corresponding components one for each vector in the sum: $F_{netx} = F_{1x} + F_{2x}$, and $F_{nety} = F_{1y} + F_{2y}$. In our example we have $F_{netx} = -2.60 + 2.00 = -0.60$ N, and $F_{nety} = 1.50 + 0.00 = 1.50$ N. We then have that $F_{net} = (F_{netx}^{\ 2} + F_{nety}^{\ 2})^{1/2} = 1.62$ N, and $\tan\theta_{net} = F_{nety} / F_{netx}$, $\theta_{net} = 112^o$ [Note here that $\theta_{net} = \tan^{-1} (F_{nety} / F_{netx}) + 180^o$, because $F_{netx} < 0$].

Graphically, the addition of two vectors ***A*** and ***B*** is carried as follows: vector ***B*** is *translated* (i.e. slides without changes in its magnitude and direction) until its tail overlaps with the head of ***A***; the sum is the arrow going from the tail of ***A*** to the head of the translated ***B***. [Note: The sum of vectors is commutative, so instead of translating ***B***, you could have translated ***A***, the sum then would be arrow going from the tail of ***B*** to the head of translated ***A***]
The sequence in Fig.2 below shows you the steps followed when finding the sum, using the graphical method, of the two forces in our example: i) The two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ acting simultaneously are shown in Fig 2a, remember the scale is 2.0 cm = 1.0 N; ii) Now we translate $\boldsymbol{F_1}$

until its tail overlaps the head of $\boldsymbol{F_2}$, and iii) The sum is the arrow going from the tail of the vector that was not translated, $\boldsymbol{F_2}$,which here is the origin, to the tip of the translated $\boldsymbol{F_1}$, Fig 2b.



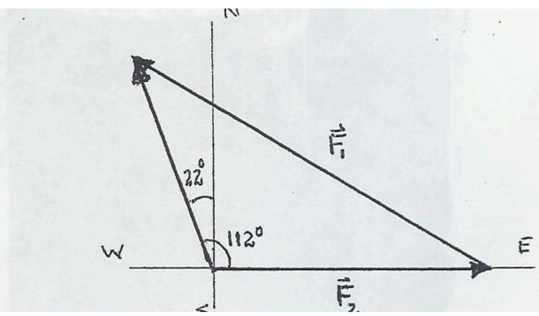


Fig. 2a. Vectors shown with common origin. Scale used is 1.0 N = 2.0 cm.

Fig 2b. Vector $\boldsymbol{F_1}$ translated so that its tail overlaps head of $\boldsymbol{F_2}$. The sum is the arrow going from the tail of $\boldsymbol{F_2}$ to the head of $\boldsymbol{F_1}$.

We measure the length of the arrow representing the sum in Fig 2b, about 3.2 cm, and remembering that our scale is 2.0 cm = 1.0 N, we determine that the magnitude of the net force is 1.6 N, and with the use of the protractor we measure the angle and determine that the net force points in the direction 22° East of North. Notice that the accuracy of the graphical method is limited by the least count of the ruler, 0.1 cm, and the protractor, 1°.

Experimental Procedure.-

1. Using the graphical and analytical methods, add the displacement vectors $\boldsymbol{d_1}$, which has magnitude 6.0 m and is directed to the East, and $\boldsymbol{d_2}$, with magnitude 8.0 m and directed to the North. For the graphical method, use a transformation scale of 1.0 m = 1.0 cm. Remember that, for the analytical method you do not need to use any transformation scale.

2. Repeat steps 1 for the case of two forces, $\boldsymbol{F_1}$ with magnitude of 5.0 N at 40° and $\boldsymbol{F_2}$ with magnitude of 5.0 N at 160°. We leave it to your common sense to choose appropriate transformation scale for the use of the graphical method; again, for the analytic method, you do not need any transformation scale.

3. Solve the following problem using the graphical and analytical methods: In going to her class a student goes through the following displacements: i) 100 m North, ii) 200 m at 45° S of E, and iii) 300 m at 30° W of N. Find the magnitude and direction of her total displacement. [Note: When adding three or more vectors graphically, you can use the *polygon method*: place the second vector so its tail overlaps the head of the first, then the third vector so that its tail overlaps the head of the second, etc. The sum is the arrow going from the tail of the first to the tip of the last one].

Note: For the graphical method, make sure to include your graphs.